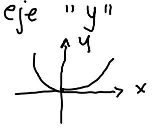
UNIDAD 9 SERIES DE FOURIER

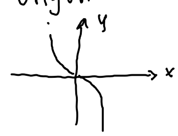
Función Original
$f(x)$

se sustituye $(-x)$

$f(-x) = f(x)$
Es par y simetrica al eje "y"



$f(-x) = -f(x)$
Es impar y simetrica al origen



No se cumple
No hay simetria

Propiedades

par * par = par
impar * impar = par
par * impar = impar

1

$\cos n\pi = (-1)^n$ $\qquad -\cos n\pi = -(-1)^{n+1}$

$\operatorname{Sen} n\pi = 0$ $\qquad \operatorname{Sen}(-n\pi) = -\operatorname{Sen} n\pi$

$\cos(0) = 1$ $\qquad \operatorname{Sen}(0) = 0$ $\qquad \operatorname{sen} \frac{\pi}{2} = 1$

$\int \cos x\,dx = \operatorname{sen} x$ $\qquad \int \operatorname{Sen} x\,dx = -\cos x$

$$\int \operatorname{Sen} nx\,dx = \frac{1}{n}\int n \operatorname{Sen} nx\,dx = -\frac{1}{n}\cos nx$$

Ejemplo.

$$f(x) = \begin{cases} \square & -\pi < x < 0 \\ \triangle & 0 < x < \pi \end{cases}$$

Periodo 2π

2

$$\int_{-\pi}^{\pi} f(x)\,dx = \int_{-\pi}^{0} \square\,dx + \int_{0}^{\pi} \Delta\,dx$$ Fourier Completa.

Par o Impar Fourier → Medio Rango.

$$\int_{0}^{\pi} f(x)\,dx = \int_{0}^{\pi} \Delta\,dx$$

3

SERIE FOURIER GENERALIZADA PARA PERIODO 2π

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \operatorname{Cos} nx + b_n \operatorname{Sen} nx)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \operatorname{Cos} nx\,dx \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \operatorname{Seno} nx\,dx$$

MEDIO RANGO.

IMPAR – SENOIDAL.

$$a_0 = 0$$
$$a_n = 0$$
$$b_n = \frac{2}{\pi}\int_{0}^{\pi} f(x) \operatorname{Sen} nx\,dx$$

PAR – COSENOIDAL

$$a_0 = \frac{1}{\pi}\int_{0}^{\pi} f(x)\,dx$$
$$a_n = \frac{1}{\pi}\int_{0}^{\pi} f(x) \operatorname{Cos} nx\,dx$$
$$b_n = 0$$

4

SERIE FOURIER GENERALIZADA PARA PERIODO $T=2L$

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L}+b_n \operatorname{Sen}\frac{n\pi x}{L}\right]$$

$$a_0=\frac{1}{L}\int_{-L}^{L} f(x)\,dx \qquad a_n=\frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n=\frac{1}{L}\int_{-L}^{L} f(x)\operatorname{Sen}\frac{n\pi x}{L}\,dx$$

Media Rango impar – senoidal

$a_0=0$
$a_n=0$

$$b_n=\frac{2}{L}\int_{0}^{L} f(x)\operatorname{Sen}\frac{n\pi x}{L}\,dx$$

5